$$\tan^{-1}\left(\frac{v_1}{k}\right) - kC_A t = \cos^{-1}\left\{ e^{C_A x}\cos\left[\tan^{-1}\left(\frac{v_1}{k}\right)\right]\right\} \tag{5,77}$$

$$t = \frac{1}{kC_A}\left(\tan^{-1}\left(\frac{v_1}{k}\right) - \cos^{-1}\left\{ e^{C_A x}\cos\left[\tan^{-1}\left(\frac{v_1}{k}\right)\right]\right\}\right) \tag{5,78}$$

$$t_2 = \frac{1}{\sqrt{C_F C_A}}\left(\tan^{-1}\left(\sqrt{\frac{C_A}{C_F}}\,v_1\right) - \cos^{-1}\left\{ e^{C_A s_H}\cos\left[\tan^{-1}\left(\sqrt{\frac{C_A}{C_F}}\,v_1\right)\right]\right\}\right) . \tag{5,79}$$

Note that (5,76A) actually allows 4 relationships using various positive or negative cosine arguments since the cosine is an even function. We choose the form (5,76B) so the time will come out positive and (5,79) will show the proper boundary behavior as C_A and C_F are varied. Equation (5,79) is the solution to the horizontal motion. It applies to the inclined plane ramp as well as to the circular arc ramp although v_1 will be different in the two cases. It will be shown in the next chapter that v_1 derived as in the last section is within 0.026% of the exact value for the circular arc case. Because the mathematical form of (5,79) is analytic, and because t_2 is still approximately inversely proportional to v_1, then 0.026% also represents the mathematical accuracy of (5,79) in representing t_2 for the circular arc case.

Equation (5,78) is complex enough that it should be checked further. We shall examine its behavior for three conditions,

1) $x \to 0$

2) $C_F \to 0$ zero wheel/axle friction, only air resistance

3) $C_A \to 0$ zero air resistance, only wheel/axle friction.

For condition 1), we can see that as x approaches zero the exponential term goes to 1 and the arctan functions cancel giving $t = 0$. For condition 2), we write (5,76B) as follows:

$$\cos\left[\tan^{-1}\left(\frac{v_1}{k}\right)\right]e^{C_A x} = \cos(kC_A t)\cos\left[\tan^{-1}\left(\frac{v_1}{k}\right)\right] + \sin(kC_A t)\sin\left[\tan^{-1}\left(\frac{v_1}{k}\right)\right] \tag{5,80}$$

$$e^{C_A x} = \cos(kC_A t) + \sin(kC_A t)\frac{\sin\left[\tan^{-1}\left(\frac{v_1}{k}\right)\right]}{\cos\left[\tan^{-1}\left(\frac{v_1}{k}\right)\right]} \tag{5,81}$$

$$e^{C_A x} = \cos(kC_A t) + \sin(kC_A t)\tan\left[\tan^{-1}\left(\frac{v_1}{k}\right)\right] \tag{5,82}$$

$$e^{C_A x} = \cos(kC_A t) + \frac{v_1}{k}\sin(kC_A t) \tag{5,83}$$